

# A short and broad introduction of the Recommendations and Advice

## A short and broad introduction of the Recommendations and Advice

*Civil engineering construction projects require to justify the work regarding the risks identified upon technical baselines, which are usually based on the current regulation or chosen by the construction manager. Then, the engineers must conduct design and execution studies with a level of detail that depends on the complexity and advancement of the project. Moreover, for civil engineering, it is a matter of studying the structural aspects considering the many possible interactions according to their predominance.*

### Structural performance

Modeling and finite elements are tools for evaluating the performance of civil engineering structures either in the design phase or verification of existing structures. These tools can be useful for analyzing new structures, but also for analyzing strengthening or structural recovery in existing ones. Thus, computational models and finite elements are first and foremost an answer to a contemporary need and not an end in itself.

“Structural performance” describes the capacity for a structure to comply with the requirements it was designed for, which are divided into three categories:

- structural safety, which ensures the structure’s resistance to the expected loads under normal circumstances as well as exceptional ones,
- serviceability, which ensures that the structure can remain in use,
- durability, which describes the capacity of the structure to remain in a state of structural safety and serviceability under given conditions of maintenance for a predefined period.

The structural performance evaluation of new or existing structures rely on the reliability theory and can be conducted using different approaches:

- deterministic (all the data are supposedly known,)
- probabilistic (the uncertainties are intrinsic and represented by random variables or functions,)
- semi-probabilistic (relying on the notions of limit state and partial safety factors.)

### Structural analysis

In the field of civil engineering, the evaluation of the structural performance is handled using a semi-probabilistic approach. It relies on the limit state method as proposed in the Eurocodes, which can help solve most of the construction cases. One can then talk about structural analysis, which is a fundamental concept of the Eurocodes.

Structural analysis is conducted by creating models, which are the results of an engineer’s reflection and assumptions, taking into consideration the available data (by default, stating the most probable hypotheses and verifying them afterward): it is all about “well-stating the problem” to solve and defining the system to study.



**Structural analysis is a process in which the engineer establishes a simplified representation of the structure that replicates as effectively as possible its mechanical behavior.**

### Modeling

According to the Eurocode, structural modeling must imply calculations that are conducted using appropriate computational models including the related variables. It is expected that structural models can predict the structural behavior within an acceptable range of precision. Moreover, the models must be adequate to the considered limit-states throughout all the steps of the project. The structural models used must be based on a theory and an established practice and must be verified experimentally if needed.

Modeling is then an approach or even a procedure, that consists of establishing a model. According to its objectives and the means used, modeling can be handled using distinct methods. It is all about representing an object, a real phenomenon, or a phenomenon extracted from its environment and simplifying it using an isolated system that follows a concept or a theory. It is developed starting from the acting physical phenomena, which means that it incorporates physical laws. Thus, the constitutive laws are formulated under mathematical equations compatible with each material and each structural element of the overall structure. This leads to solving complex continuum mechanics systems of equations written as partial differential equations.

The main peculiarity of civil engineering calculations compared to other engineering fields is the need to consider all the phases of construction such as the clearance and filling works, the concrete work of the foundations, attaching braces, tensioning, or anchoring cables. Modeling transcribes these events in the static language in various ways: by the continuation of computation with internal constraints (“initial constraints”,) by changing the stiffness of some elements (go from a zero value to a positive one,) by getting rid of or modifying the nature of some supports, by changing the points of application of loads, etc.

“Modern structures” in civil engineering (built after the second industrial revolution) present favorable features to their structural analysis: homogeneous geometry, repeated patterns, qualitative construction materials (conformed to the “product standards”.) These structures are however affected by multiple factors that are hardly or cannot be controlled: environmental factors such as the type of soil, the quality of the construction itself, as the structure might be built-in zones of limited access or under unfavorable meteorological conditions (e.g. the sensitivity of wooden

elements to humidity, which can impact the dimensions and the mechanical features of the element under normal conditions by creating gaps or shrinkage, or steel elements that expand when subjected to high temperatures...,) and the heterogeneous aging of their various elements.

To face the issues related to designing and the equilibrium verification of real structures, the structural analysis relies on creating models, which means creating simplifications of the real structure: design of materials, ideal kinematic assumptions (perfect connections, bilateral, independent from the reaction forces,) and even arbitrary choices about what factors are involved in the equilibrium of the structure (by not considering some elements or by neglecting the involvement of some components on the structural behavior.) These simplifications imply that there is only a contingent relationship with the real structures in their environment. Consequently, the output results from the modeling must always be analyzed critically while keeping in mind the explicit assumptions and the less explicit ones that were made: firstly, isolating the system from its environment, then selecting a mathematical model and a solving method...

The exact solutions are extremely rare (only a few textbook cases,) it becomes a necessity to solve real problems using approximated methods:

- either analytically: for instance, using Strength of Materials formulations for which geometrical considerations lead to a simplification of the problem from 3D to 2D (average surface of plates and shells) or even 1D (average fiber of beams) ...
- or numerically: FE methods, finite difference methods...

To validate the obtained results, one can compare them to experimental values. It is important to note that different sources of error exist in this comparison such as:

- modeling errors: coming from the modification of the real problem into a simplified one,
- discretization errors: due to the numerical resolution of the problem,
- measurement errors: related to the equipment and the measuring process.

The obtained results should, ideally, be compared to the experimental measurements that can also be filled with errors... Moreover, it is possible to combine two families of methods: FE for beams, finite differences for plates...

**The simplification should allow to solve the problem to evaluate numerically the effects (efforts, stresses, displacements) of different load cases on the structure. However, it should fit as well as possible the behavior of the real structure.**



*From the problem to its approximated solution*

## Finite Element (FE) Method

The FE method is a modeling tool that allows us to solve numerically problems that are set using partial differential equations. The FE method can be applied to civil engineering design since modern construction materials such as steel or concrete allow to work in the continuous framework, which is the underlying assumption of the FE formulations. Thus, it is implied that before using this method, the engineer had a preliminary reflection about it and is aware of the modeling procedure.

The FE method is a general numerical method allowing to handle real-life (“industrial”) problems from various fields (solid mechanics, fluid mechanics, thermodynamics...) in continuous frameworks, static or dynamic, linear or non-linear, as well as coupled problems: thermo-mechanical behaviors (structures subjected to fire,) fluid-structure (vibrations of a tank containing liquids,) chemical-mechanical (considering corrosion of rebars in reinforced concrete)... The surge for the FE method is closely related to the improvement of computer science since the 1980s and led to the creation of numerous software. This guideline covers only the use of the FE method, however, this is not the only structural analysis method: besides some traditional approaches such as the graphical statics, the use of graphs and tables or analog techniques on models (recognized by the structural Eurocodes,) there exist much more tools (spectral methods, Boundary Element Methods (BEM), finite difference, etc..) Some methods are still being researched, but the FE approach is the most general and the most used in the field.

Comprehensively, the principle behind the FE method can be summarized as follows: the domain of analysis, originally complex, is divided (discretized) into a wide range of subdomains with simple shapes (**finite elements**) from which points are identified (nodes): the **meshing** is now defined. The approximated solution is calculated only at the nodes (discrete solutions.) The solution at any other point can be recovered by interpolating the values at the nodes.

More specifically, the different steps of the FE method are:

1. Discretizing the structure in several nodes linked by elements that have well-known behaviors. Establishing the column displacement vector  $D$  containing the  $N$  degrees of freedom of the structure. The degrees of freedom are the possible displacements (translations and rotations according to the model) of the structure. There are the unknowns of the problem that belongs to the  $R^N$  space.
2. Establishing the behavior of the elements. It can be divided into two parts:
  1. Determining the displacement forces  $f_{ed}$ : the forces applied by the nodes linked to the element on this element to inflict its displacements  $d_e$ . The relation is established in a local coordinate system specific to each element, then transposed to the global coordinate system:  $F_{ed}=K_e \cdot D_e$ . One can now go back to the relation given in 1 p 8, but for an educational purpose, it might be preferable to handle separately the issue of the combined behavior of the elements (which is addressed by picking the solution in some kind of behavioral library.)
  2. Determining the constraint forces  $f_{eb}$ : the forces applied by the nodes on an element directly loaded, to prevent any displacement. The nodes, in this case, play a virtual support role for the element, and one can calculate the reaction forces in a local coordinate system of the element  $f_{eb}$ , then in the global coordinate system  $F_{eb}$ .

3. Assembling. Establishing the structure's stiffness matrix and the forces applied by the nodes onto the elements according to their loading. This process is conducted by summation, in the  $R^N$  space for vectors and  $R^N \times R^N$  space for matrices, of the elementary stiffness matrices and the constraint force matrices:  $K = \sum K_e$  and  $F_B = \sum F_{Be}$ .
4. Writing and solving the equation of displacements, which is the equilibrium equation of the nodes. The nodes are applying forces onto the elements, which leads to a displacement  $D$  (KD) and constraint forces, namely reaction forces, as a consequence of the loads directly applied to the elements ( $F_B$ .) The nodes can be subjected to external forces that would be directly applied to the nodes ( $F_N$ .) such as reaction forces at the support for instance. The general equation writes  $KD = F$  (with  $F = F_N - F_B$ .)
5. Solving: displacement calculations:  $D = K^{-1}F$ . In practice, one must distinguish the degrees of freedom with known values (the supports) from the others. Let  $D_1$  denote the unknown degrees of freedom and  $D_2$  the known degrees of freedom. One then organizes the working space in a way that the unknowns are first.

The general equation then reads:



In this equation,  $F_1$  and  $D_2$  are known. One must then calculate



then  $F_2$ .

6. Knowing the displacements, it is possible to determine the efforts in the elements by using the fundamental behavior equations of each element.



*Geometry and meshing (triangular finite elements and nodes.)*

This guideline describes the available tools often used in the computational FE software in statics and dynamics. To save space and be as clear as possible, we have chosen not to introduce any problem related to interstitial flow despite their great interest in geotechnical work. Regarding the difficulty of modeling, the geotechnical field (also covered by the FE method) raises challenges, on the one hand, analogous to the issues met in dynamics of structures (choosing the time discretization,) and on the other hand, elastoplastic issues (heterogeneous permeability, boundary between saturated and non-saturated zones, etc.).



**The FE method is a tool for numerical calculations. The modelling prior to solving can rely on all the principles of the preexisting structural analysis.**

🔄Révision #5

★Créé 18 September 2023 14:42:48 par Paul Terrasson Duvernon

✍Mis à jour 10 October 2023 09:02:34 par Paul Terrasson Duvernon